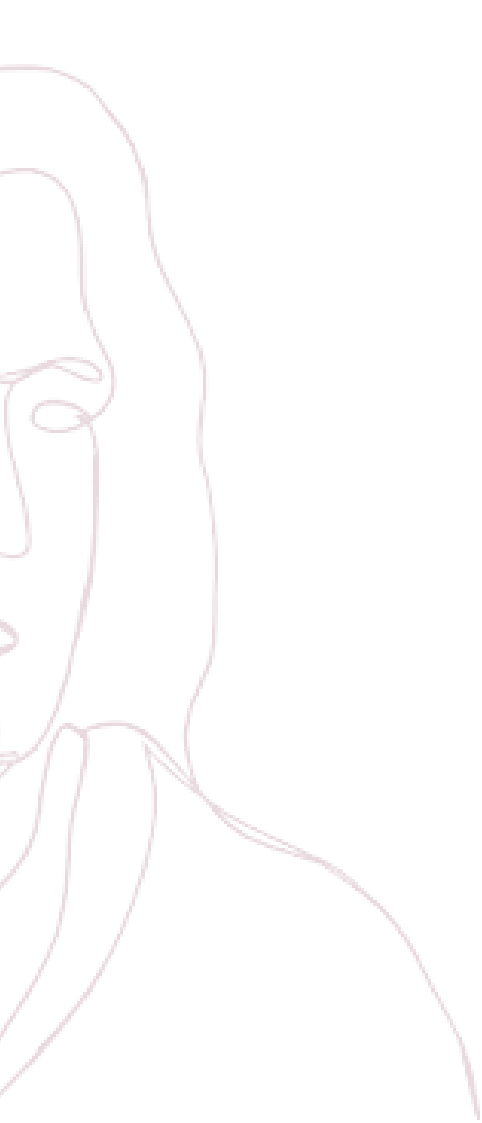




Inflation and Index Numbers Workshop

Statistics I · Lecture 3

Today

- Inflation: CPI and HICP, tests for composite indices (if not covered in Lecture 2)
- Workshop: Topic 1 exercises, from  to 

Inflation

Consumer Price Index

To measure inflation, we use a Laspeyres Price Index, usually known as CPI (Consumer Price Index).

To do that, the National Institute of Statistics (or the equivalent national statistical office) defines a basket, which contains a certain mix of goods and services, that is to be representative of the consumption behavior of the population.

In Portugal , the index has base in 2016, and, as stated above, follows a LPI formulation with small technical adjustments. This index has a monthly frequency, *i.e.* it is computed every month.

Consumer Price Index

To create a price index, you need to follow at least 3 steps.

1. Determine representative basket: Survey families   
2. Survey retail stores to find out sale prices. 
3. Compute the index with the collected data. 
 1. Compute the cost of the  with the prices at the base period.
 2. Compute the cost of the same  with the prices of every successive period.

Application

A Labor union leader is complaining that the average salary in 2022 was 1500, and in 2025 it is 1650. This increase, they say, is not enough to maintain the workers' purchasing power, because of inflation. The data for inflation is the following:

Year	2022	2023	2024	2025
CPI	113	119	122.2	124.8

1. The year with the largest increase in prices was 2024?
2. The average growth rate for the wages between 2022 and 2025 was _____?
3. Is the union leader right?

Application

For the first question, we can compute the chain or link index:

Year	2022	2023	2024	2025
CPI	113	119	122.2	124.8
CPI_{link}	—	105.3	102.7	102.1

So the answer is: The year with the largest price increase was 2023.

Just in case there were doubts, an example:

$$CPI_{link|2024} = \frac{122.2}{119} \times 100 = 102.7$$

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Application

We know salary in 2022 was 1500, and in 2025 1650. If the growth rate was constant, say δ_w we would have:

$$1650 = 1500(1 + \delta_w)^3$$

Or conversely

$$\frac{1650}{1500} = (1 + \delta_w)^3$$

or

$$(1.1)^{1/3} = 1 + \delta_w$$

$$1.0323 - 1 = \delta_w \Rightarrow \delta_w = 0.0323 = 3.23\%$$

Summarizing

Remember this relationship we saw before?

$$I_{t|b}^q = \frac{I_{t|b}^v}{I_{t|b}^p}$$

Real equals nominal over CPI.

This, dividing by CPI a nominal quantity, is known as *to deflate* by the CPI.

About the Test for Factor Reversal for Composite Index Numbers

Note that neither Laspeyres nor Paasche pass the test for **factor reversal** ([see here](#)),

$$VI_{t|0} \neq LPI_{t|0} \times LQI_{t|0} \quad VI_{t|0} \neq PPI_{t|0} \times PQI_{t|0}$$

But we can verify that:

$$LPI_{t|0} \times PQI_{t|0} = IV_{t|0} \quad PPI_{t|0} \times LQI_{t|0} = IV_{t|0}$$

About the Test for Factor Reversal for Composite Index Numbers

(... continuation)

Example (do the rest as homework):

$$LPI_{t|0} \times PQI_{t|0} = \frac{\sum_{k=1}^m p_t^k q_0^k}{\sum_{k=1}^m p_0^k q_0^k} \frac{\sum_{k=1}^m p_t^k q_t^k}{\sum_{k=1}^m p_t^k q_0^k} = \frac{\sum_{k=1}^m p_t^k q_t^k}{\sum_{k=1}^m p_0^k q_0^k}$$

Note that this last item is exactly $IV_{t|0}$.

You can verify, using the same procedure, that Fisher's index does indeed pass the factor reversal test.

About the Factor Reversal Test for Composite Index Numbers

$$\begin{aligned}
 FPI_{t|0} \times FQI_{t|0} &= \\
 \sqrt{LPI_{t|0} \times PPI_{t|0}} \times \sqrt{LQI_{t|0} \times PQI_{t|0}} \\
 &= \sqrt{LPI_{t|0} \times PPI_{t|0} \times LQI_{t|0} \times PQI_{t|0}} \\
 &= \sqrt{LPI_{t|0} \times PQI_{t|0} \times LQI_{t|0} \times PPI_{t|0}} \\
 &= \sqrt{IV_{t|0} \times IV_{t|0}} = \sqrt{IV_{t|0}^2} = IV_{t|0}
 \end{aligned}$$

About the Time Reversal Test for Composite Index Numbers

Laspeyres and Paasche do not pass the time Reversal test, let's see an example:

$$LPI_{t|0} \times LPI_{0|t} = \frac{\sum_{k=1}^m p_t^k q_0^k}{\sum_{k=1}^m p_0^k q_0^k} \frac{\sum_{k=1}^m p_0^k q_t^k}{\sum_{k=1}^m p_t^k q_t^k} \neq 1$$

You can do the same for Paasche, and for the quantities index numbers.

About the Time Reversal Test for Composite Index Numbers

Fisher on the other hand...

$$\begin{aligned}
 FPI_{t|0} \times FPI_{0|t} &= \sqrt{LPI_{t|0} PPI_{t|0}} \sqrt{LPI_{0|t} PPI_{0|t}} \\
 &= \sqrt{\frac{\sum_{k=1}^m p_t^k q_0^k}{\sum_{k=1}^m p_0^k q_0^k} \frac{\sum_{k=1}^m p_t^k q_t^k}{\sum_{k=1}^m p_0^k q_t^k}} \sqrt{\frac{\sum_{k=1}^m p_0^k q_t^k}{\sum_{k=1}^m p_t^k q_t^k} \frac{\sum_{k=1}^m p_0^k q_0^k}{\sum_{k=1}^m p_t^k q_0^k}} \\
 &= \sqrt{\frac{\sum_{k=1}^m p_t^k q_0^k}{\sum_{k=1}^m p_0^k q_0^k} \frac{\sum_{k=1}^m p_t^k q_t^k}{\sum_{k=1}^m p_0^k q_t^k} \frac{\sum_{k=1}^m p_0^k q_t^k}{\sum_{k=1}^m p_t^k q_t^k} \frac{\sum_{k=1}^m p_0^k q_0^k}{\sum_{k=1}^m p_t^k q_0^k}}
 \end{aligned}$$

About the Time Reversal Test for Composite Index Numbers

Fisher on the other hand...

$$\begin{aligned}
 FPI_{t|0} \times FPI_{0|t} &= \sqrt{LPI_{t|0} PPI_{t|0}} \sqrt{LPI_{0|t} PPI_{0|t}} \\
 &= \sqrt{\frac{\sum_{k=1}^m p_t^k q_0^k}{\sum_{k=1}^m p_0^k q_0^k} \frac{\sum_{k=1}^m p_t^k q_t^k}{\sum_{k=1}^m p_0^k q_t^k}} \sqrt{\frac{\sum_{k=1}^m p_0^k q_t^k}{\sum_{k=1}^m p_t^k q_t^k} \frac{\sum_{k=1}^m p_0^k q_0^k}{\sum_{k=1}^m p_t^k q_0^k}} \\
 &= \sqrt{\frac{\sum_{k=1}^m p_t^k q_0^k}{\sum_{k=1}^m p_0^k q_0^k} \frac{\sum_{k=1}^m p_t^k q_t^k}{\sum_{k=1}^m p_0^k q_t^k} \frac{\sum_{k=1}^m p_0^k q_t^k}{\sum_{k=1}^m p_t^k q_t^k} \frac{\sum_{k=1}^m p_0^k q_0^k}{\sum_{k=1}^m p_t^k q_0^k}} \\
 FPI_{t|0} \times FPI_{0|t} &= 1
 \end{aligned}$$

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About the Circularity Test for Composite Index Numbers

None, not even Fisher's Index, passes the circularity test.

Homework: Verify it.

Example

Public Transportation	Year 1		Year 2	
	Trips	Price per trip	Trips	Price Index (Base: year 1)
Bus	1,763,521	1.15	1,875,345	108.70
Metro	3,148,350	1.2	3,396,138	112.50

Example

1. According to LPI for year 2, prices for public transportation grew 11.17%.
2. Real change, according to Paasche, for public transportation was 7.35%.
3. The composite index for change in expenditures with public transportation in year 2 compared the expenditure in year 1 is what type of change? (real, nominal, quantity, prices?) How much was it?

Example

According to LPI for year 2, prices for public transportation grew 11.17%.

First, we need to find p_2 for bus and metro.

$$108.7 = \frac{p_2^{bus}}{1.15} \times 100$$

$$\text{Or } p_2^{bus} = 1.25$$

$$112.5 = \frac{p_2^{metro}}{1.2}$$

$$\text{Or } p_2^{metro} = 1.35$$

Example

	Year 1	Year 2
Public Transportation	N Trips	Ticket price
Bus	1,763,521	1.15
Metro	3,148,350	1.2

Example

$$LPI_{2|1} = \frac{\sum_{k=1}^m p_2^k q_1^k}{\sum_{k=1}^m p_1^k q_1^k}$$

$$LPI_{2|1} = \frac{1,763,521 \times 1.25 + 3,148,350 \times 1.35}{1,763,521 \times 1.15 + 3,148,350 \times 1.2}$$

$$LPI_{2|1} = 1.1117$$

The *basket* in year 2 is 11.17% more expensive than in year 1.

$$LPI_{2|1} \times 100 - 100 = 11.17\%$$

Example

Real change, according to Paasche, for public transportation was 7.35%.

$$PQI_{2|1} = \frac{\sum_{k=1}^m p_2^k q_2^k}{\sum_{k=1}^m p_2^k q_1^k}$$

$$PQI_{2|1} = \frac{1,875,345 \times 1.25 + 3,396,138 \times 1.35}{1,763,521 \times 1.25 + 3,148,350 \times 1.35}$$

$$PQI_{2|1} = 1.0735$$

Indeed, according to the Paasche index, in year 2 the amount of public transportation trips was 7.35% larger than in year 1.

$$PQI_{2|1} \times 100 - 100 = 7.35\%$$

Example

The composite index for change in expenditures with public transportation in year 2 compared the expenditure in year 1 is what type of change? (real, nominal, quantity, prices?) How much was it?

The value index for public transportation is a *nominal* index (price times quantity).

Remember:

$$VI_{t|0} = LPI_{t|0} \times PQI_{t|0}$$

Example

$$VI_{2|1} = LPI_{2|1} \times PQI_{2|1} = 1.1117 \times 1.0735 \approx 1.1934$$

The value of public transportation trips grew 19.34% from year 1 to year 2. This includes the effect of an increase in prices and quantities.

? Inflation · Question 1

The Consumer Price Index is constructed as a:

A. Paasche quantity index

B. simple index number

C. Fisher index

D. Laspeyres price index

 **D.** The basket is fixed at a base period, which is exactly a Laspeyres price index (with small technical adjustments).

? Inflation · Question 2

Your nominal wage rises 10% while the CPI rises 12%. Your purchasing power:

A. rose about 10%

B. did not change

C. rose 2%

D. fell about 1.8%

 **D.** Real change = $\frac{1.10}{1.12} - 1 \approx -0.0179$, a fall of about 1.8%. Prices grew faster than the wage.



Inflation · Question 3

A worker earned 1500 three years ago and earns 1650 today. Over the same period the CPI went from 100 to 112.

Did the worker gain or lose purchasing power, and by how much?



Inflation · Solution

Nominal wage growth: $\frac{1650}{1500} - 1 = 0.10$, that is 10%. Price growth: $\frac{112}{100} - 1 = 0.12$, that is 12%.

$$\delta^q = \frac{1 + \delta^v}{1 + \delta^p} - 1 = \frac{1.10}{1.12} - 1 \approx -0.0179$$

The worker **lost** about 1.8% of purchasing power: the wage grew, but prices grew faster. 

Workshop: Index Numbers

Workshop

Today we practice everything from Topic 1. The exercises go from easy to hard:

-  warm-up: one formula, one step
-  standard: two or three steps
-  exam level: this is what you will find in the midterm
-  stretch: you need to combine several things

Try each one before the solution shows up. 10 minutes each, more or less.

Workshop · Quick round 1

An index number is used:

- A. to measure the change in prices
 - B. to measure the change in consumed quantities
 - C. to measure the change of a variable over time, or against a reference
 - D. to measure the changes in the demand
-  **C.** Prices and quantities are only two particular cases. We used it for stock prices, sales, water consumption...

Workshop · Quick round 2

The most adequate average to compute the average growth rate of, for example, prices is the:

A. median

B. arithmetic mean

C. harmonic mean

D. geometric mean

☒ **D.** Growth compounds, so we multiply the factors $(1 + \delta_t)$ and take the k -th root: $\bar{\delta} = (\prod_t (1 + \delta_t))^{1/k} - 1$.

Workshop · Quick round 3

The Consumer Price Index (CPI) is computed using:

- A. the Laspeyres formula
 - B. the Paasche formula
 - C. the average of Laspeyres and Paasche
 - D. the Fisher formula
-  **A.** The basket is fixed in the base period, and only the prices change.

Workshop · Exercise 1

Consider the following sales data, in thousands of euros, for a given company:

Year	2020	2021	2022	2023	2024	2025
Sales	500	650	600	800	750	950

- (a)** Build the index series with fixed base in 2020.
- (b)** Study the evolution of sales between consecutive years.
- (c)** What was the growth rate between 2022 and 2024? And the average growth rate for the whole period?



Workshop · Exercise 1 · Solution

(a) and (b)

Year	2020	2021	2022	2023	2024	2025
Index (base 2020)	100	130	120	160	150	190
Chain index		130	92.3	133.3	93.8	126.7

Sales grew 30%, fell 7.7%, grew 33.3%, fell 6.2% and grew 26.7%. Over the whole period they grew 90%.

(c) $\delta_{2024|2022} = \frac{750}{600} - 1 = 0.25$, that is 25%.

$\bar{\delta} = \left(\frac{950}{500}\right)^{1/5} - 1 = 1.9^{0.2} - 1 \approx 0.137$, about 13.7% per year.

Workshop · Exercise 2

For some country, water consumption between 2017 and 2025 evolved according to the following simple index, base 2022:

2017	2018	2019	2020	2021	2022	2023	2024	2025
75.6	78.5	82.5	88.1	93.8	100	104.8	107.7	107.9

- (a) What was the growth of water consumption between 2019 and 2021?
- (b) Find the chain index for 2019 and 2020.
- (c) Change the base to 2017. What is $I_{2025|2017}$?
- (d) Find the growth rate and the average growth rate for the whole period.



Workshop · Exercise 2 · Solution

- (a) $\frac{93.8}{82.5} = 1.137$, so consumption grew 13.7%. Note we do not need the base, only the ratio.
- (b) $I_{2019}^{chain} = \frac{82.5}{78.5} \times 100 = 105.1$ and $I_{2020}^{chain} = \frac{88.1}{82.5} \times 100 = 106.8$
- (c) $I_{2025|2017} = \frac{107.9}{75.6} \times 100 = 142.7$
- (d) Consumption grew 42.7% in 8 years, so $\bar{\delta} = 1.427^{1/8} - 1 \approx 0.0455$, about 4.55% per year.

Workshop · Exercise 3

Wayne Industries reported sales using the following chain index:

2019	2020	2021	2022	2023	2024	2025
120	125	135	145	150	160	165

- (a) What was the average growth rate between 2019 and 2025?
- (b) What was the growth rate between 2018 and 2025?
- (c) What was the average **semi-annual** growth rate between 2021 and 2024?



Workshop · Exercise 3 · Solution

(a) From 2019 to 2025 we multiply the six links **after** 2019:

$$1.25 \times 1.35 \times 1.45 \times 1.50 \times 1.60 \times 1.65 \approx 9.69$$

$\bar{\delta} = 9.69^{1/6} - 1 \approx 0.4601$, about 46.01% per year.

(b) Now the 120 counts too, it is the growth from 2018 to 2019: $1.20 \times 9.69 \approx 11.628$, a growth of 1062.8% 😬

(c) $1.45 \times 1.50 \times 1.60 = 3.48$ in 3 years, which are 6 semesters: $3.48^{1/6} - 1 \approx 0.2310$, about 23.10% per semester.

Workshop · Exercise 4

A given company presented the following information:

Year	2020	2021	2022	2023	2024	2025
Growth rate of sales		20%	15%	20%	10%	25%
Price Index (base 2022)	81.17	89.29	100	115	128.8	148.12

- (a)** Find the average growth rate of (nominal) sales for the whole period.
- (b)** Find the annual **real** growth rate of sales. When was it negative?
- (c)** How do you interpret the price index for 2020 and 2021?

✓ Workshop · Exercise 4 · Solution

(a) $\bar{\delta} = (1.2 \times 1.15 \times 1.2 \times 1.1 \times 1.25)^{1/5} - 1 \approx 0.1789$, about 17.89% per year.

(b) Build the sales index with base 2022, and deflate it with $I^q = \frac{I^v}{I^p}$:

Year	2020	2021	2022	2023	2024	2025
Sales index	72.46	86.96	100	120	132	165
Real sales index	89.27	97.39	100	104.35	102.48	111.40

Real growth was negative only in 2024: $\frac{102.48}{104.35} - 1 \approx -1.79\%$

(c) From 2020 to 2022 prices grew $\frac{100}{81.17} - 1 \approx 23.20\%$, and from 2021 to 2022 they grew 11.99%.

Workshop · Exercise 5

Consider the following data on nominal wages for the workers of Watto's Junkyard, and the inflation rate in Tatooine:

Year	2021	2022	2023	2024	2025
Wage Index (base 2022)	93.5	100	104.3	103.8	109.7
Inflation rate	3.0%	2.4%	2.8%	2.3%	3.0%

- (a) What was the nominal growth of wages from 2021 to 2025?
- (b) Build the chain index for wages.
- (c) In which year did wages have the largest **real** increase?



Workshop · Exercise 5 · Solution

(a) $\frac{109.7}{93.5} - 1 \approx 0.1733$, so 17.33%.

(b) and (c) For the real change, $\delta^q = \frac{1+\delta^v}{1+\delta^p} - 1$:

Year	2022	2023	2024	2025
Chain index	106.95	104.30	99.52	105.68
Real change	4.45%	1.46%	-2.72%	2.61%

The largest real increase was in 2022. In 2024 the workers lost purchasing power twice: the wage fell, and prices went up.

Workshop · Exercise 6

Consider the following sales data, in thousands of euros:

Year	2020	2021	2022	2023	2024	2025
Sales	300	250	a	240	b	340

- (a) Find the average growth rate of sales between 2020 and 2025.
- (b) You know that $I_{2022|2020} = 67$, and that the growth rate between 2023 and 2024 was **double** the one between 2022 and 2023. Find a and b .
- (c) What average growth rate is needed to triple sales between 2025 and 2029?
- (d) With an average growth rate of 10%, how many years would sales need to triple?



Workshop · Exercise 6 · Solution

(a) $\left(\frac{340}{300}\right)^{1/5} - 1 \approx 0.0253$, about 2.53% per year. We do not need a nor b !

(b) $a = 0.67 \times 300 = 201$. Then $\delta_{2023} = \frac{240}{201} - 1 \approx 0.1940$, so $\delta_{2024} \approx 0.3881$ and $b = 240 \times 1.3881 \approx 333.1$

(c) $(1 + \bar{\delta})^4 = 3 \Rightarrow \bar{\delta} = 3^{1/4} - 1 \approx 0.3161$, about 31.61% per year.

(d) $1.1^n = 3 \Rightarrow n = \frac{\ln 3}{\ln 1.1} \approx 11.5$, so 12 years (not 6!).

Workshop · Exercise 7

The table has prices and quantities of two goods   , before and after some austerity measures. Between the two periods there are 3 years.

	Price before	Quantity before	Price after	Quantity after
Bread (units) 	0.20	420	0.25	360
Milk (lts) 	0.70	360	0.80	336

- (a) What was the average annual growth rate of the price of bread?
- (b) Find the Laspeyres and Paasche **quantity** indices, and Fisher's.
- (c) Were the real and nominal changes in spending similar?



Workshop · Exercise 7 · Solution

(a) $\left(\frac{0.25}{0.20}\right)^{1/3} - 1 \approx 0.0772$, about 7.72% per year.

(b) $LQI = \frac{0.2 \times 360 + 0.7 \times 336}{0.2 \times 420 + 0.7 \times 360} = \frac{307.2}{336} \approx 0.914$ and $PQI = \frac{0.25 \times 360 + 0.8 \times 336}{0.25 \times 420 + 0.8 \times 360} = \frac{358.8}{393} \approx 0.913$

$FQI = \sqrt{0.914 \times 0.913} \approx 0.9135$, quantities fell about 8.65%.

(c) $IV = \frac{358.8}{336} \approx 1.068$: spending grew 6.8% in nominal terms, while the quantities fell 8.65%. Not similar at all, prices did all the work.